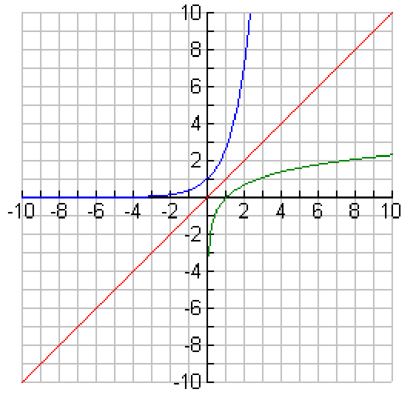
1.4

Logarithmic Functions and Solving

The inverse of an exponential function $f(x) = b^x$ with base b is the logarithmic function with base b, $f^{-1}(x) = \log_b x$.



The domain of the logarithm function is $x > 0$.

Domain of a logarithm function = range of the exponential function = $(0,\infty)$ and the range of a logarithm function = domain of exponential function $(-\infty,\infty)$.

Changing Between Logarithmic and Exponential Form

If $x > 0$ and $0 < b \neq 1$, then **$y = \log_b(x)$ (read as "y is the logarithm to the base b of x")**

if and only if $b^y = x$.

Evaluating Logarithms

Examples:

b) $\log_3 \sqrt{3} = \frac{1}{2}$, *because* $3^{\frac{1}{2}} = \sqrt{3}$

c) $\log_5 \frac{1}{25} = -2,\ \textit{because}\ 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

d) $\log_4 1 = 0,\ \textit{because}\ 4^0 = 1$

e) $\log_7 7 = 1,\ \textit{because}\ 7^1 = 7$

Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y,

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

Evaluating Logarithmic and Exponential Expressions

Examples:

a) $\log_2 8 = \log_2 2^3 = 3$

b) $\log_3 \sqrt{3} = \log_3 3^{\frac{1}{2}} = \frac{1}{2}$

c) $6^{\log_6 11} = 11$

Basic Properties of Common Logarithms

Let x and y be real numbers with x > 0.

$\log 1 = 0$ because $10^0 = 1$

$\log 10 = 1$ because $10^1 = 10$

$10^{\log x} = x$ because $\log x = \log x$

log 10^y = y because $10^y = 10^y$

Evaluating Logarithmic & Exponential Expressions—Base 10

Examples:

$a)$ $\log 100 = \log_{10} 100 = 2,$ $because\, 10^2 = 100$

b) $\log \sqrt[5]{10} = \log 10^{\frac{1}{5}} = \frac{1}{5}$

c) $\log \frac{1}{1000} = \log \frac{1}{10^3} = \log 10^{-3} = -3$

d) $10^{\log 6} = 6$

Evaluating Common Logarithms with a Calculator

Examples:

a) $\log 34.5 = 1.537\ldots,$ $because$ $10^{1.537\ldots} = 34.5$

b) $\log 0.43 = -0.366\ldots,$ $because$ $10^{-0.366\ldots} = 0.43$

c) $\log(-3)$ is undefined because there is no real number y such that $10^y = -3.$

Solving Simple Logarithmic Equations

Examples: Solve each equation by changing it to exponential form.

a) $\log x = 3$

Change to exponential form, $x = 10^3 = 1000$

b) $\log_2 x = 5$

Change to exponential form, $x = 2^5 = 32$

Basic Properties of Natural Logarithms

Let x and y be real numbers with x > 0

ln 1 = 0 because e^{0} **=** 1.

ln e = 1 because e^{1} = e.

$e^{\ln x}$ = x because ln x = ln x.

ln e^{y} = y because $e^{y} = e^{y}$.

Evaluating Logarithmic & Exponential Expressions Base – e

Examples:

a) $\ln\sqrt{e} = \log_e \sqrt{e} = \frac{1}{2}$, $\textit{because } e^{\frac{1}{2}} = \sqrt{e}$

b) $\ln e^5 = \log_e e^5 = 5$

c) $e^{\ln 4} = 4$

Evaluating Natural Logarithms with a Calculator

Examples:

a) $\ln 23.5 = 3.157\ldots$, $\textit{because } e^{3.157\ldots} = 23.5$

b) $\ln 0.48 = -0.733\ldots$, $\textit{because } e^{-0.733\ldots} = 0.48$

c) $\ln(-5)$ is undefined because there is no real number y such that $e^{y} = -5$.

One to one properties:

For any exponential function $f(x) = b^x$,

- If $b^u = b^v$, then $u = v$.

For any logarithmic function $f(x) = \log_b x$,

- If $\log_b u = \log_b v$, then $u = v$.

Solving Exponential Equations:

Solve: $20\left(\frac{1}{2}\right)^{\frac{x}{3}} = 5$

$$20\left(\frac{1}{2}\right)^{\frac{x}{3}} = 5$$

$$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \left(\frac{1}{2}\right)^{2}$$

$$\frac{x}{3} = 2 \qquad x = 6$$

See example 2 pg. 321.

Solving Logarithmic Equations:

Solve: $\log x^2 = 2$.

$\log x^2 = 2 \quad \rightarrow \quad 10^2 = x^2 \quad \rightarrow \quad x = 10$ or -10

See methods 1 & 3 also pg. 321- 322.

Solve: $\ln(3x - 2) + \ln(x - 1) = 2\ln x$.

$\ln(3x - 2) + \ln(x - 1) = 2\ln x$

$\ln[(3x - 2)(x - 1)] = \ln x^2$

$(3x - 2)(x - 1) = x^2$

$3x^2 - 3x - 2x + 2 = x^2$

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = \frac{1}{2}$ or $x = 2$ check domain $x \neq \frac{1}{2}$ so $x = 2$ is the only solution.

We can also solve this by graphing, setting the original equation equal to 0 and finding the x-intercepts.

Solve: $\ln(3x - 2) + \ln(x - 1) = 2\ln x$.

$\ln(3x - 2) + \ln(x - 1) - 2\ln x = 0$

x-intercept is at (2, 0).

<u>Newton's Law of Cooling :</u>

$T(t) = T_m + (T_o - T_m)\, e^{-kt}$

T_m = the temperature of the surrounding medium

T_o = initial temperature of the object.

See example 7 pg. 326.